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Mark Scheme (Results)

June 2011

GCE Mechanics M4 (6680) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

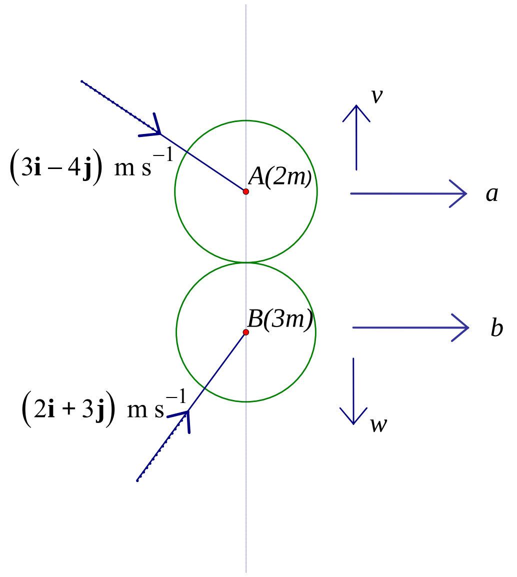
   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

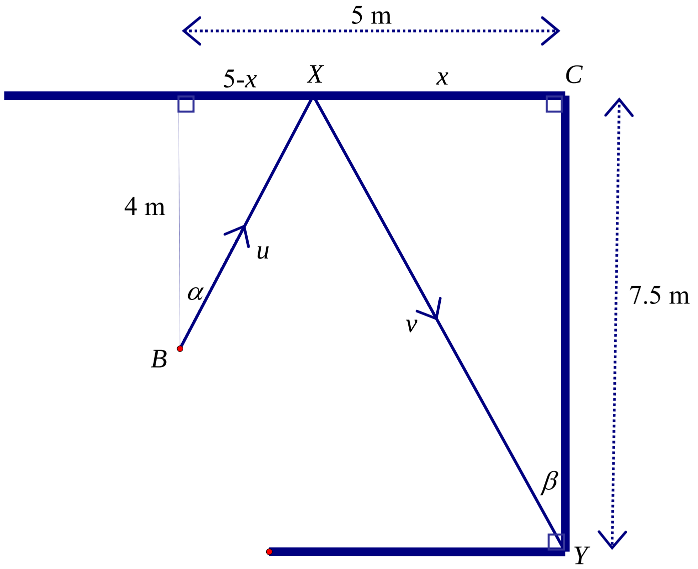
3. Abbreviations

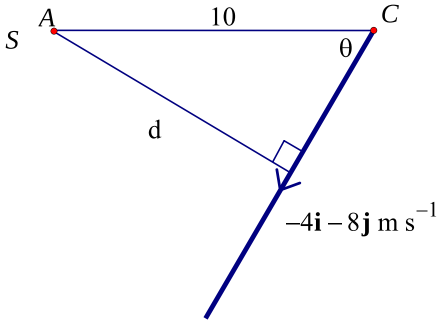
   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

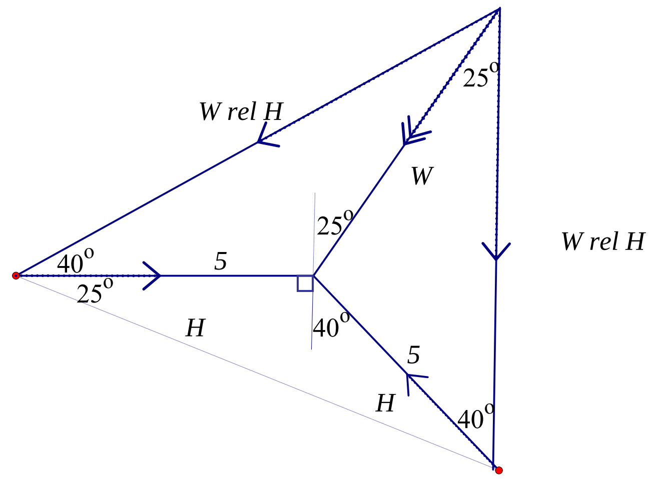
   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper
   - $\sqsubset$ The second mark is dependent on gaining the first mark

June 2011
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.		
	$\leftrightarrow a=3$ & $b=2$	B1
	b Conservation of linear momentum: $-4\times2+3\times3 \;=2v-3w(=1)$	M1A1
	Restitution : $v+w=\mathrm{e}\times7\ \ (=3)$	M1A1
	Solve the simultaneous equations	DM1
	giving $v=2$ and $w=1$	A1
	KE lost $=\frac{1}{2}\times2m\times((16+9)-(4-9))+\frac{1}{2}\times3m\times((9+4)-(1-4))$	M1A1
	$=24m\ \ (\mathrm{J})$	A1
		10

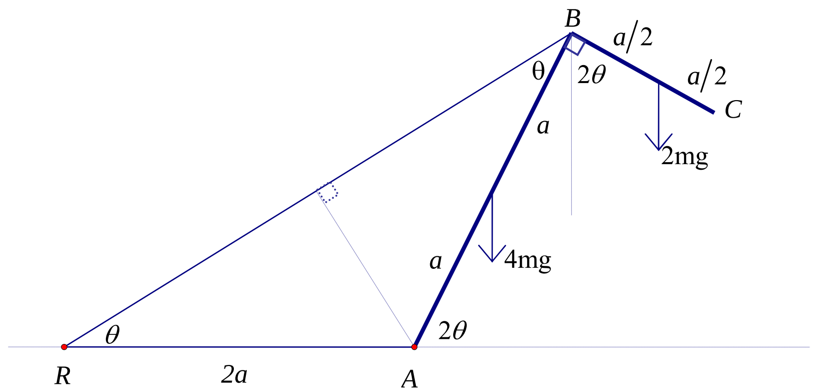
Question Number	Scheme	Marks
2.		
	At X: $\leftrightarrow$ $u\sin\alpha = v\sin\beta$	M1A1
	$\updownarrow$ $v\cos\beta = eu\cos\alpha$	M1A1
	$4v\cos\beta = 3u\cos\alpha$	
	Eliminate u & v by dividing: $\frac{\tan\alpha}{3} = \frac{\tan\beta}{4}$	M1
	Substitute for the trig ratios: $\frac{5-x}{3\times 4} = \frac{x}{4\times 7.5}$	DM1A1
	Solve for x: $37.5 - 7.5x = 3x$	DM1
	$x = 3.57\ (\text{m})$ or better, $\frac{25}{7}$	A1
		9
3. (a)	Velocity of C relative to S $= (8\mathbf{i}+u\mathbf{j}) - (12\mathbf{i}+16\mathbf{j})$	M1
	$= (-4\mathbf{i} + (u-16)\mathbf{j})(\text{m s}^{-1})$	A1
		(2)
(b) (i)	C intercepts S $\Rightarrow$ relative velocity is parallel to **i**.	
	$\Rightarrow u - 16 = 0,\ u = 16$	M1A1
		(2)
(ii)	10 km at 4 km h^{-1} takes 2.5 hours, so 2.30pm	M1A1
		(2)

Question Number	Scheme	Marks
(c)	$u=8$, relative velocity $=-4\mathbf{i}-8\mathbf{j}$.	B1
		
	Correct distance identified	B1
	Using velocity: $\tan\theta=\frac{8}{4}=2\Rightarrow\sin\theta=\frac{2}{\sqrt{5}}$	
	Using distance: $\sin\theta=\frac{d}{10}=\frac{2}{\sqrt{5}}$,	M1A1
	$\mathrm{d}=\frac{20}{\sqrt{5}}=4\sqrt{5}=8.9\ (\mathrm{km})$	A1 (5)
		11

Question Number	Scheme	Marks
4. **(a)**	 2 vector triangles with a common side …..correct and drawn on a single diagram Wind is from bearing 025°, (N 25° E)	M1 A1 A1 (3)
(b)	$\frac{5}{\sin 25^{\circ}} = \frac{W}{\sin 40^{\circ}}$ (ft on their 25°) $W = \frac{5 \times \sin 40^{\circ}}{\sin 25^{\circ}} = 7.6\ \left(\text{km h}^{-1}\right)$	M1A1ft M1A1 (4)

Question Number	Scheme	Marks
5. **(a)**	Need an equation linking speed and displacement, so $mv\dfrac{dv}{dx} = -(a+bv^2)$ Separating the variables: $\int \dfrac{6v}{a+bv^2}dv = \int -1dx$ Integrating : $\dfrac{3}{b}\ln(a+bv^2) = -x+(C)$ $X = \dfrac{3}{b}\left[\ln(a+bU^2)-\ln(a)\right] = \dfrac{3}{b}\ln\left[1+\dfrac{bU^2}{a}\right]$ ** as required	M1A1 M1 A1 M1A1 (6)
(b)	Equation connecting v and t: $6\dfrac{dv}{dt} = -(12+3v^2)$ Separate the variables: $\int \dfrac{-6}{12+3v^2}dv = \int 1dt$ $\int_U^0 \dfrac{-2}{4+v^2}\mathrm{d}v = \int_0^U \dfrac{2}{4+v^2}\mathrm{d}v = T$ $T = \dfrac{2}{2}\tan^{-1}\dfrac{U}{2} = \tan^{-1}\dfrac{U}{2}(\mathrm{s})$	M1 M1, A1 M1 A1 (5) **11**

Question Number	Scheme	Marks
6. **(a)**	Using F = ma: $4\dfrac{d^2x}{dt^2} = -9x - 12v$	M1A1
	$= -9x - 12\dfrac{dx}{dt}$	M1
	Hence $4\dfrac{d^2x}{dt^2} + 12\dfrac{dx}{dt} + 9x = 0$ **	A1
		(4)
(b)	Auxiliary eqn : $4m^2 + 12m + 9 = 0$,	B1
	$(2m+3)^2 = 0, m = -3/2,\ \lambda = 3/2$	B1
	$t = 0, x = 4 \Rightarrow B = 4$	B1
	$t = 0,\ \dot{x} = e^{-\lambda t}\left(-\lambda\left(At + B\right) + A\right) = 0 \Rightarrow -6 + A = 0,\ A = 6$	B1
		(4)
(c)	$\dot{x} = e^{-\frac{3}{2}t}(-\frac{3}{2}(6t+4)+6) = -9te^{-\frac{3}{2}t}$	M1A1
	$\ddot{x} = e^{-\frac{3}{2}t}(-9-(-9t)\times\frac{3}{2})$,	M1
	so acceleration = 0 when $t = 2/3$	
	at which time, $v = -6e^{-1}$, so max speed $= 6/e \approx 2.21$ m s^{-1} (3sf)	A1, A1
		(5) **13**

Question Number	Scheme	Marks
7. **(a)**	 $\text{BR} = 2\times 2a\cos\theta = 4a\cos\theta$ $\text{EPE} = 3mg\dfrac{(4a\cos\theta)^2}{2\times 2a}$ $= 12mga\cos^2\theta = 6mga + 6mga\cos 2\theta$ GPE: taking AR as the level of zero GPE, GPE = GPE of AB + GPE of BC $= 4mg\times a\sin 2\theta + 2mg\left(2a\sin 2\theta - a/2\cos 2\theta\right)$ $= 8mga\sin 2\theta - mga\cos 2\theta$ $\Rightarrow$ Total $V = 8mga\sin 2\theta + 5mga\cos 2\theta + \text{constant}$, as required. **	 B1 M1 A1 M1+M1 A1 A1 (7)
(b)	$\dfrac{dV}{d\theta} = 16mga\cos 2\theta - 10mga\sin 2\theta$ $\dfrac{dV}{d\theta} = 0 \Rightarrow 10\sin 2\theta = 16\cos 2\theta$ $\Rightarrow \tan 2\theta = \dfrac{8}{5} \Rightarrow \theta = 0.51$ radians (29.0°) Or: $8mga\sin 2\theta + 5mga\cos 2\theta = \sqrt{89}mga\cos(2\theta-\alpha)$, $\tan\alpha = \dfrac{8}{5}$ t. pts when $2\theta - \alpha = n\pi \Rightarrow \theta = 0.51$ rads.	M1 A1 M1 A1 (4) M1A1 M1A1
(c)	$\dfrac{d^2V}{d\theta^2} = -32mga\sin 2\theta - 20mga\cos 2\theta$ $\theta = 0.51 \Rightarrow \dfrac{d^2V}{d\theta^2} < 0$, equilibrium is unstable. cso Or: $2\theta - \alpha = 0 \Rightarrow \cos(2\theta - \alpha) = 1$ Max value $\Rightarrow$ equilibrium is unstable	M1 M1A1 (3) **14**

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